Erratum for Denjoy Minimal Sets Are Far From Affine

ALEC N. KERCHEVAL

Dept. of Mathematics, Florida State University, Tallahassee, FL 32308-4510 (e-mail: kercheva@math.fsu.edu)

(Received April 2010)

Abstract. Theorem 2 of [1] is corrected by adding a C^2 bound to the hypotheses.

Consider the circle \mathbf{R}/\mathbf{Z} with coordinates [0,1). Let $L \subset [0,1)$ be a compact interval and for $k \geq 2$ let $\mathcal{I} = \{I_1, \ldots, I_k\}$ be a collection of pairwise disjoint compact intervals with union $I \subset L$. Define $\mathcal{S}^r(\mathcal{I}, L)$ to be the set of C^r functions $S: I \to L$ such that |S'| > 1 on I, and for each $j = 1, \ldots, k$, $S[I_j] = L$. For such S, define its $nonlinearity \mathcal{N}(S) \equiv \max_j \sup\{\log(S'(x)/S'(y)) : x, y \in I_j\}$.

Any $S \in \mathcal{S}^r(\mathcal{I}, L)$ has a unique maximal invariant (Cantor) set $C_S = \{x \in I : S^n(x) \in I \text{ for all } n > 0\}$. A Cantor set is C^1 -minimal if it is the minimal set of some C^1 diffeomorphism of the circle. The following appears as Theorem 2 in [1].

THEOREM 1. Let I_1, \ldots, I_k, L be compact intervals as above. Then there exists $\epsilon > 0$ (depending only on $\{|I_j|/|L| : j = 1, \ldots, k\}$) such that if $S \in \mathcal{S}^2(\mathcal{I}, L)$ and $\mathcal{N}(S) < \epsilon$ then C_S is not C^1 -minimal.

A. Portela correctly points out that the proof in [1] only proves the following slightly weaker statement:

THEOREM 2. Let I_1, \ldots, I_k, L be compact intervals as above, and let M > 0. Then there exists $\epsilon > 0$ (depending only on $\{|I_j|/|L| : j = 1, \ldots, k\}$ and M) such that if $S \in \mathcal{S}^2(\mathcal{I}, L), |S''| < M$, and $\mathcal{N}(S) < \epsilon$ then C_S is not C^1 -minimal.

It still follows immediately that C^1 -minimal Cantor sets are not C^2 -nearly affine:

COROLLARY 1. Let I_1, \ldots, I_k, L be given as above. Then there exists $\epsilon > 0$ such that for all $A, S \in \mathcal{S}^r(\mathcal{I}, L)$, if A is locally affine and $||S - A||_{C^2} < \epsilon$, then C_S is not C^1 -minimal.

References

 A. N. Kercheval. Denjoy minimal sets are far from affine. Ergodic Theory and Dynamical Systems, 22, 2002, 1803–1812.